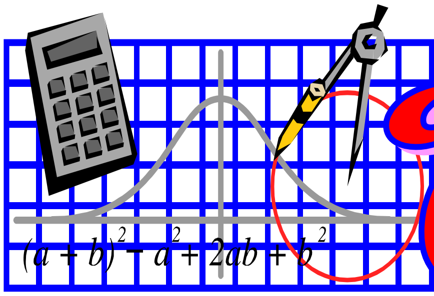

Algebra Connections

Mr. Breitsprecher's Edition | October 3, 2005 | Web: www.clubtnt.org/my_algebra

FUN With Formulas

Before working with formulas, let's look at some units of measurement – many formulas will be based on units. In the US, we use feet, yards, and miles.

12 inches = 1 foot; 3 feet = 1 yard; 5,280 feet = 1 mile. Units used in formulas must be the same – so changing from one unit into another equivalency is important.

Here are the conversions:

- **Feet to Inches**. Number of feet * 12
- **Inches to Feet**. Number of inches/12
- **Yards to Feet**. Number of yards * 3
- **Miles to Feet**. Number miles * 5,280

Metrics are used throughout the world and are universally accepted in the sciences. These measures are all based on a system of 10's – we will look at them in another edition of ***Mr. Breitsprecher's Algebra Connections.***

Often, proportions are the best way to solve measurement problems. Recall that a proportion is a statement that 2 ratios are equal. The key is to use the same units in each ratio, identify 3 of the figures in the proportion, and solve for the fourth. The **cross product** is an important shortcut to help us set up these problems (see previous edition ***Mr. Breitsprecher's Algebra Connections*** for more on this)

Once we are comfortable working with units and performing conversions, we are ready to work with formulas. There is nothing new here – identify the known quantities and solve for the unknown using the algebraic procedures reviewed in class. Here are some common formulas that will be used in many practical situations and in Algebra.

Perimeter

The distance around the outside of a given area, or 2 dimensional shape, is called the perimeter. Think of it as the total length of the border around that shape. Here are some common perimeter formulas.

Perimeter of a Triangle. A closed figure with three straight sides is called a triangle. We can refer to each side as side 1, side 2, and side 3. The perimeter is the sum of these sides. The formula is: $\mathbf{P = s_1+s_2+s_3}$

Perimeter of a Square. A 4-sided, closed figure with each corner measuring 90 degrees (right angle) is a square. While we could express the formula for the perimeter as the sum of the 4 sides, this would look awkward – it is easier to express it in terms of multiplication. The formula is: **P = 4s**

Perimeter of a Rectangle. A 4-sided, closed figure with each corner measuring 90 degrees (right angle) is a rectangle. We could also express its perimeter as the sum of the sides, but that would also look awkward – it is easier to express it in terms of multiplication. The formula is: **P = 2l (length)+2w (width).**

Perimeter of a Circle. A circle has a perimeter, but we call it the circumference. This word derives from circumstance – think of the situations around you. There are no sides to measure, so we use the diameter (D) – a line drawn through the center point of a circle that touches both sides of the figure. Sometimes, we are only given the radius (r), which is half the diameter (D/2). We need a special mathematical figure for this formula, π. For most purposes, we can approximate this with 3.14 – though it really will go on and on forever without repeating. When convenient, we can also use the fraction 22/7. The formula for the circumference of a circle is: $\mathbf{C=\pi D}$

Area

A measurement of how many 2-dimensional units (squares) a particular object or surface covers is called the area. Think of it as the flat space a shape occupies. It is measured in square units, usually square inches, square centimeters, square feet, or square miles, and so forth. Imagine a floor covered with square tiles that are each 1-foot x 1 foot. If we count the tiles, we have the area in square feet.

Area of a Triangle. Think of the area of a triangle as ½ that of an imaginary rectangle that includes the triangle. We need to know the base (b) or bottom of the triangle. We also need the height (h), the perpendicular distance from the base to the top angle of the triangle. The formula is: **$A = 1/2\ bh$**

Area of a Square. Because of their square corners (90 degrees) and opposites sides, the area of a square is the same formula as the area of a rectangle. It is the products of 2 sides (remember, they are equal). The formula is:
$A = l$ (length) $*$ w (width)

Area of a Rectangle. For an explanation, please see above. The formula is: **$A = l$ (length) $*$ w (width).**

Area of a Circle. The area of a circle is based on the radius (r), which is ½ the diameter (D/2). The formula is **$A=\pi r^2$**

Volume

Area is a 2 dimensional, or flat measurement. Volume is a 3-dimensional measurement. Think of it as being based on the measurement across, front-to-back, and up-and-down. Imagine counting how many 1-inch square sugar cubes would fit in a box – that is what volume does. Volume is always measure in cubed units (cubic inches, cubic feet, cubic centimeters, and so forth).

Volume of Pyramid (Three Dimensional Triangle). The volume of a pyramid with a square base is one-third the area of the base (b) multiplied by the height (h). The formula is: **$V= 1/3(s^2)*h$**

Volume of a Square/Cube. The 90-degree corners and equal sides make this one straightforward. Note that a 3-dimensional square is called a cube. The formula is: **$V = s^3$**

Volume of a Rectangle. This one is based on the same idea as the volume of a square – it's the product of the length, width, and height. The formula is:
$V=l$(length) $*w$(width)$*h$(height)

Volume of a Sphere. This expression, like the area, is based on the radius (r), which is ½ the diameter (D). We also need to use pi. The formula is: **$V = 4/3(\pi r^3)$**

Volume of a Cylinder. Think of this shape as a 3-dimensional cross between a rectangular box and a sphere. The volume is based on the radius (r), the height (h) and pi. The formula is: **$V = \pi r^2 h$**

Volume of a Cone. This shape is a 3-dimensional cross between a triangle and a circle and is based on the radius (r), the height (h) and pi. The formula is: **$V = 1/3(\pi r^2 h)$**

Temperature Conversions

Both Fahrenheit and Celsius are based on the freezing and boiling points of water. Fahrenheit uses 32 for freezing and 212 for boiling. Celsius uses 0 for freezing and 100 for boiling. It is probably easiest to work with Celsius, but we typically use Fahrenheit.

Fahrenheit to Celsius. Subtract 32 from the F temperature, multiply by 5 and divide by 9. The formula is: **$C= (5/9)\ (F-32)$**

Celsius to Fahrenheit. Multiple the Celsius temperatures by 9, divide by 5, and add 32. The formula is: **$F = (9/5)C+32$**

More Useful Formulas

Distance. The formula for calculating distances is based on rate (speed expressed as a ratio of distance per unit of time) and time. Be sure that the distance is expressed in the same unit as the rate. Read these problems carefully – identify the variables that are know, express all variables based on the same units, and then plug the known variables (with the same units) into the formula and solve for the unknown. The formula is:
$d = r$ (rate)$*t$ (time)

Percent. Always representing a ratio of some percentage to 100, we calculate this based on the base and rate. If we express ratios in terms of 100, we make comparisons more meaningful. When 2 ratios are involved, solve percentage problems with proportions using the ***cross product.*** The formula is:
p (percentage) $= b$ (base) $* r$ (rate)

Pythagorean Theorem. A special case of a triangle is when 1 angle equals 90 degrees – this is a called right triangle. The Pythagorean Theorem states that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse (the side directly opposite of the 90 degree or right angle). Many distance and height problems can be solved by applying this theorem. The formula is
c^2 (hypotenuse) $= a^2$ (side adjacent right angle) $+ b^2$ (other side adjacent right angle)

Simple Interest. We expect to pay people for the privilege of borrowing money – we pay back more than we borrow. The "extra" amount is "interest." There are different ways to calculate this; some are complex. Simple interest is computed only on the original amount of a loan. To calculate this amount, we need to know how much is borrowed (principal), the annual interest rate (expressed as a decimal, **NOT** as a percent), and the amount of time the money will be borrowed in years (if given in months, divide by 12 to express as part of a year). The formula is:
I (interest) $= P$ (principal)$*r$ (annual rate) $* t$ (time in years)

Geometric & Other Formulas